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Engineering Notes

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Relative Spacecraft Motion in a Central Force Field

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Nomenclature

e = eccentricity (dimensionless)
f = true anomaly, rad
$\mathbf{h}$ = specific angular momentum, m^2/s
i = inclination, rad
p = semilatus rectum, m
$\boldsymbol{r}$ = radius vector, m
r = magnitude of vector $\boldsymbol{r}$
t = time, s
u = argument of the latitude, rad
$\boldsymbol{v}$ = velocity vector, m/s
$\boldsymbol{\alpha}$ = angular velocity, rad/s
$\tilde{\boldsymbol{\alpha}}$ = skew-symmetric tensor (matrix) associated with vector $\boldsymbol{\alpha}$
μ = Earth gravitational parameter, m^3/s^2
Ω = right ascension of the ascending node, rad
ω = argument of perigee, rad
$\triangleq$ = by definition, it equals to...

Introduction

SPACECRAFT formations are a principal feature of many space missions. Formation flight reduces both cost and complexity of spacecraft. The main goal of the present research is to derive more accurate models for the relative orbital motion to increase the motion predictability for the relative trajectory [1].

Recent papers [2–5] deal with new solutions to the orbital relative motion problem; instead of considering the classical linearized equations of motion, the exact solution in a matrix form is obtained. Our new approach provides a method to obtain arbitrary high-order approximations to the relative spacecraft motion by using Cartesian coordinates combined with classical orbital elements.

The present paper shows that various matrix expressions for the solution to the relative orbital motion may be written in a much simpler form, which was introduced for the first time in [6,7]. Instead of using the orbital elements as constants of motion, this novel approach is based on a tensorial frame-independent form of the solution to the relative orbital dynamics.

The results deduced in the case of the relative spacecraft motion in a gravitational field from a particle are generalized to the case of an arbitrary central force field. An interesting geometric visualization for the relative orbital motion in a central force field is presented.

Problem Formulation

Consider a body having a Keplerian motion around an attraction center O. Its trajectory is a conical section (elliptic, parabolic, or hyperbolic). This will be the reference satellite, named the Chief. The satellite to be observed will be named the Deputy. It orbits the same attraction center. The main problem in orbital relative dynamics is to obtain the law of motion of the Deputy with respect to the Chief.

Traditionally, the following reference frames are used in studying the motion of a Deputy satellite [2]: 1) an inertial frame OXYZ, named Earth-centered inertial (ECI); 2) a rotating reference frame OPQR, with OP pointing in the direction of the Chief satellite, OR having the direction of the specific angular momentum of the spacecraft (normal to the trajectory plane), and OQ completing the right-handed frame of the system; and 3) a noninertial reference frame Cxyz that has the origin at the Chief center of mass and whose axes are parallel to the axes of OPQR. This frame will be referred to as local-vertical-local-horizontal (LVLH).

The main problem in orbital relative dynamics is to determine the motion of the Deputy satellite with respect to the LVLH noninertial reference frame associated with the Chief satellite.

The Deputy motion with respect to the LVLH reference frame is modeled by the initial value problem:

$$\begin{cases} \ddot{\boldsymbol{r}} + 2\boldsymbol{\alpha}\times\dot{\boldsymbol{r}} + \boldsymbol{\alpha}\times(\boldsymbol{\alpha}\times\boldsymbol{r}) + \dot{\boldsymbol{\alpha}}\times\boldsymbol{r} + \dfrac{\mu}{|\boldsymbol{r}_C+\boldsymbol{r}|^3}(\boldsymbol{r}_C+\boldsymbol{r}) - \dfrac{\mu}{r_C^3}\boldsymbol{r}_C = 0 \\ \boldsymbol{r}(t_0) = \Delta\boldsymbol{r}, \qquad \dot{\boldsymbol{r}}(t_0) = \Delta\boldsymbol{v} \end{cases} \tag{1}$$

where $\boldsymbol{\alpha}$ represents the angular velocity of the rotating frame OPQR, $\mu > 0$ the Earth gravitational parameter, t_0 the initial moment of time, $\boldsymbol{r}_C$ the position vector of the Chief with respect to an inertial reference frame whose origin is at the attraction center, $\Delta\boldsymbol{r}$ the initial relative position vector and $\Delta\boldsymbol{v}$ the initial relative velocity of the Deputy with respect to LVLH. In LVLH, $\boldsymbol{r}_C$ is a vector that has constant direction and variable magnitude. By denoting $\mathbf{h}_C$ the Chief specific angular momentum and f_C its true anomaly, it follows that:

$$\boldsymbol{\alpha} = \dot{f}_C \frac{\mathbf{h}_C}{h_C} \tag{2}$$

A matrix transformation between the inertial frame OXYZ and the rotating frame OPQR appears in Balaji and Tatnall [2] and Gurfil and Kasdin [3]. It links the equations of motion expressed with respect to these reference frames. The procedure consists in the transformation from the inertial frame OXYZ to the rotating noninertial reference frame OPQR, by means of three successive rotations [2].

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The transformation matrix from OXYZ to OPQR is

$$[A]_{I-R} = \begin{bmatrix} \cos u_C & \sin u_C & 0 \\ -\sin u_C & \cos u_C & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i_C & \sin i_C \\ 0 & -\sin i_C & \cos i_C \end{bmatrix} \times \begin{bmatrix} \cos \Omega_C & \sin \Omega_C & 0 \\ -\sin \Omega_C & \cos \Omega_C & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

where $u_C = \omega_c + f_C$ is the argument of the latitude, i_C the inclination, and Ω_C the right ascension of the ascension node of the Chief.

The coordinates of the Deputy spacecraft with respect to the rotating noninertial OPQR reference frame are [2]

$$\begin{bmatrix} P_D \\ Q_D \\ R_D \end{bmatrix} = [A]_{I-R} \begin{bmatrix} X_D \\ Y_D \\ Z_D \end{bmatrix} \tag{4}$$

where $[\boldsymbol{r}_D]_I = \begin{bmatrix} X_D & Y_D & Z_D \end{bmatrix}^T$ are the coordinates of the Deputy with respect to the ECI inertial reference frame OXYZ. One may remark that $[\boldsymbol{r}_D]_I$ is the solution to the initial value problem:

$$\ddot{\boldsymbol{r}} + \frac{\mu}{r^3}\boldsymbol{r} = \boldsymbol{0}, \qquad \boldsymbol{r}(t_0) = \boldsymbol{r}_C^{\boldsymbol{0}} + \Delta\boldsymbol{r}$$
$$\dot{\boldsymbol{r}}(t_0) = \boldsymbol{v}_C^0 + \Delta\boldsymbol{v} + \boldsymbol{\alpha}_0 \times \Delta\boldsymbol{r}, \qquad \boldsymbol{\alpha}_0 = \boldsymbol{\alpha}(t_0) \tag{5}$$

where $\boldsymbol{r}_C^{\boldsymbol{0}}$ is the position vector of the Chief at $t = t_0$ and $\boldsymbol{v}_C^0$ represents the absolute velocity of the Chief at $t = t_0$.

The position vector of the Deputy with respect to the LVLH reference frame associated with the Chief may now be expressed as [2,3]

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = [A]_{I-R} \begin{bmatrix} X_D \\ Y_D \\ Z_D \end{bmatrix} - \begin{bmatrix} r_C \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

where $r_C = \frac{p_C}{1+e_C \cos f_C}$ (p_C is the Chief semilatus rectum, e_C the Chief trajectory eccentricity, and f_C the Chief true anomaly). In most papers, the reference trajectory is considered to be circular or elliptic. In fact, because no particular conditions were imposed for any of the trajectories involved, the preceding approach remains valid for any Keplerian reference and observed trajectories.

Remarks on the Matrix Solution to the Relative Spacecraft Motion

One may remark that the transformation matrix $[A]_{I-R}$ introduced in Eq. (3) is proper orthogonal.

The associated instantaneous angular velocity may be determined from [8]

$$\tilde{\boldsymbol{\alpha}} = \left(\frac{\mathrm{d}}{\mathrm{d}t}[A]_{I-R}\right)[A]_{I-R}^T \tag{7}$$

where $\tilde{\boldsymbol{\alpha}}$ is the skew-symmetric tensorial map associated with the instantaneous angular velocity $\boldsymbol{\alpha}$, usually defined as [8]: $\tilde{\boldsymbol{\alpha}}\boldsymbol{x} = \boldsymbol{\alpha} \times \boldsymbol{x}$ for any vector $\boldsymbol{x}$.

From Eqs. (7) and (3), it follows that

$$\left(\frac{\mathrm{d}}{\mathrm{d}t}[A]_{I-R}\right)[A]_{I-R}^T = \begin{bmatrix} 0 & \dot{u}_C & 0 \\ -\dot{u}_C & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{8}$$

Because $\dot{u}_C = \dot{f}_C$, the rotation rate vector $\boldsymbol{\alpha} = \boldsymbol{\alpha}(t)$, $t \geq t_0$ associated with the orthogonal matrix $[A]_{I-R}$ is

$$\begin{bmatrix} 0 \\ 0 \\ -\dot{f}_C \end{bmatrix} \stackrel{\Delta}{=} -\boldsymbol{\alpha} \tag{9}$$

Note that $\boldsymbol{\alpha} = \begin{bmatrix} 0 & 0 & \dot{f}_C \end{bmatrix}^T$ represents the instantaneous angular velocity vector of the rotating reference frame OPQR. It is expressed with respect to the rotating reference frame OPQR. It follows that

$$[A]_{I-R} = \mathbf{R}_{-\boldsymbol{\alpha}} \tag{10}$$

where $\mathbf{R}_{-\boldsymbol{\alpha}}$ is the rotation matrix (tensorial map) that models the rotation with angular velocity $-\boldsymbol{\alpha}$ [6]:

$$\mathbf{R}_{-\boldsymbol{\alpha}} = \mathbf{I}_3 - \frac{\sin[f_C(t) - f_C(t_0)]}{\alpha}\tilde{\boldsymbol{\alpha}} + \frac{1 - \cos[f_C(t) - f_C(t_0)]}{\alpha^2}\tilde{\boldsymbol{\alpha}}^2 \tag{11}$$

(f_C represents the true anomaly of the Chief trajectory). Equation (11) is a Rodrigues-like formula [8]. One may remark that matrix $[A]_{I-R}$ does not depend on the orbital elements of the Chief, as it may result from Eq. (3), but it depends only on the Chief true anomaly f_C.

The position vector of the Deputy spacecraft with respect to the LVLH reference frame may be expressed as

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{R}_{-\boldsymbol{\alpha}} \begin{bmatrix} X_D \\ Y_D \\ Z_D \end{bmatrix} - \begin{bmatrix} r_C \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

This result may be written in a tensorial coordinate-independent form [6] and applied to the relative spacecraft motion as [7]

Proposition 1: The relative law of motion of the Deputy with respect to the LVLH reference frame is given by

$$\boldsymbol{r} = \mathbf{R}_{-\boldsymbol{\alpha}(t)}\boldsymbol{r}_D - \boldsymbol{r}_C \tag{13}$$

where $\boldsymbol{r}_D$ is the solution to the initial value problem:

$$\ddot{\boldsymbol{r}} + \frac{\mu}{r^3}\boldsymbol{r} = \boldsymbol{0}, \qquad \boldsymbol{r}(t_0) = \boldsymbol{r}_C^0 + \Delta\boldsymbol{r}$$
$$\dot{\boldsymbol{r}}(t_0) = \boldsymbol{v}_C^0 + \Delta\boldsymbol{v} + \boldsymbol{\alpha}_0 \times \Delta\boldsymbol{r} \tag{14}$$

$\boldsymbol{\alpha}_0 = \boldsymbol{\alpha}(t_0)$, $\mathbf{R}_{-\boldsymbol{\alpha}}$ is given by Eq. (11), and $\boldsymbol{r}_C$ is defined as

$$\mathbf{r}_C = \frac{p_C}{1 + e_C \cos f_C(t)} \frac{\mathbf{r}_C^0}{r_C^0} \tag{15}$$

Equation (13) is the exact vectorial closed-form solution to Eq. (1). The result stands true for any reference trajectory: elliptic, parabolic, or hyperbolic. In fact, the initial value problem (1) is reduced to the study of the classic inertial Kepler problem (14). Equation (12) is reestablished in a tensorial form in Eq. (13).

Remark 1: The motion defined in Eq. (13) is a composition between a Foucault Pendulum-like motion and a rectilinear motion. A Foucault Pendulum-like motion is defined as a motion that takes place in a variable plane that has a precession with a given instantaneous angular velocity, which in this case is $-\boldsymbol{\alpha}$, around a fixed point.

Relative Motion in a Central Force Field

The results for the relative Keplerian spacecraft motion may be generalized to the relative motion in a central force field. For instance, this is the model for the relative spacecraft dynamics under the influence of the oblateness perturbation factor J_2 [9,10].

Consider that the Chief is moving in a central force field. Its motion is modeled by the inertial initial value problem:

$$\ddot{\boldsymbol{r}} + g(r)\boldsymbol{r} = \boldsymbol{0}, \qquad \boldsymbol{r}(t_0) = \boldsymbol{r}_C^0, \qquad \dot{\boldsymbol{r}}(t_0) = \boldsymbol{v}_C^0 \tag{16}$$

where g is a continuous real valued function. The trajectory is a planar curve and the motion has constant specific angular momentum $\mathbf{h}_C = \boldsymbol{r} \times \dot{\boldsymbol{r}} = \boldsymbol{r}_C^0 \times \boldsymbol{v}_C^0 \neq \boldsymbol{0}$.

As with the Keplerian relative dynamics, let us consider now the Chief and Deputy as two bodies moving under the influence of the same central force field. We consider the same reference frames as in the preceding sections. The motion of the Deputy with respect to the OPQR reference frame is modeled by the initial value problem:

$$\begin{cases} \ddot{\boldsymbol{r}} + 2\boldsymbol{\alpha}\times\dot{\boldsymbol{r}} + \boldsymbol{\alpha}\times(\boldsymbol{\alpha}\times\boldsymbol{r}) + \dot{\boldsymbol{\alpha}}\times\boldsymbol{r} + g(r)\boldsymbol{r} = 0 \\ \boldsymbol{r}(t_0) = \boldsymbol{r}_C^{\mathbf{0}} + \Delta\boldsymbol{r}, \qquad \dot{\boldsymbol{r}}(t_0) = \Delta\boldsymbol{v} + \dfrac{\boldsymbol{r}_C^0\cdot\boldsymbol{v}_C^0}{(r_C^0)^2}\boldsymbol{r}_C^0 \end{cases} \tag{17}$$

where $\boldsymbol{\alpha} = \dot{\varphi}\frac{\mathbf{h}_C}{h_C} = \frac{1}{r_C^2}\mathbf{h}_C$ represents the angular velocity of the rotating reference frame OPQR (φ is the angle between $\boldsymbol{r}_C$ and $\boldsymbol{r}_C^{\mathbf{0}}$), t_0 the initial time, $\boldsymbol{r}_C^{\mathbf{0}}$ the position vector of the Chief at $t = t_0$, $\boldsymbol{v}_C^0$ the inertial velocity of the Chief at $t = t_0$ and $\Delta\boldsymbol{r}$, $\Delta\boldsymbol{v}$ the relative position vector and the relative velocity vector of the Deputy with respect to the Chief, respectively. The next result was introduced for the first time in [11] and it offers a representation for the motion with respect to a rotating reference frame under the influence of a central force field. Its proof may also be found in [6] in the particular case of the gravitational field from a particle.

Proposition 2: The solution to the initial value problem

$$\ddot{\boldsymbol{r}} + 2\boldsymbol{\alpha}\times\dot{\boldsymbol{r}} + \boldsymbol{\alpha}\times(\boldsymbol{\alpha}\times\boldsymbol{r}) + \dot{\boldsymbol{\alpha}}\times\boldsymbol{r} + g(r)\boldsymbol{r} = \mathbf{0}, \qquad \boldsymbol{r}(t_0) = \boldsymbol{r}_0 \\ \dot{\boldsymbol{r}}(t_0) = \boldsymbol{v}_0 \tag{18}$$

is obtained by applying the operator $\mathbf{R}_{-\boldsymbol{\alpha}}$ to the solution to the initial value problem:

$$\ddot{\boldsymbol{r}} + g(r)\boldsymbol{r} = \mathbf{0}, \qquad \boldsymbol{r}(t_0) = \boldsymbol{r}_0, \qquad \dot{\boldsymbol{r}}(t_0) = \boldsymbol{v}_0 + \alpha_0\times\boldsymbol{r}_0 \tag{19}$$

where $\boldsymbol{\alpha}_0 = \boldsymbol{\alpha}(t_0)$ and $\mathbf{R}_{-\boldsymbol{\alpha}}$ is defined as the solution to the initial value problem [6]:

$$\dot{\mathbf{R}}_{-\boldsymbol{\alpha}} + \tilde{\boldsymbol{\alpha}}\mathbf{R}_{-\boldsymbol{\alpha}} = \mathbf{0}_3, \qquad \mathbf{R}_{-\boldsymbol{\alpha}}(t_0) = \mathbf{I}_3 \tag{20}$$

$$\mathbf{R}_{-\boldsymbol{\alpha}} = \mathbf{I}_3 - \sin\varphi(t)\tilde{\mathbf{u}} + [1-\cos\varphi(t)]\tilde{\mathbf{u}}^2, \qquad \tilde{\mathbf{u}} = \frac{\tilde{\boldsymbol{\alpha}}}{\alpha} \\ \varphi(t) = \int_{t_0}^{t}\alpha(\tau)\,\mathrm{d}\tau \tag{21}$$

The preceding result allows a geometrical visualization of the motion described by Eq. (18): 1) a classic central force field motion in a variable plane $\Pi(t)$, $t \geq t_0$ [it is the plane of the trajectory described by Eq. (19)]; 2) a precession of plane $\Pi(t)$ with angular velocity $-\boldsymbol{\alpha}$ around the attraction center O.

The next result is based on Proposition 2 and it generalizes the solution to the relative spacecraft motion from Eq. (13) to the case of the relative motion in a central force field [12], which is modeled by the initial value problem:

$$\begin{cases} \ddot{\boldsymbol{r}} + 2\boldsymbol{\alpha}\times\dot{\boldsymbol{r}} + \boldsymbol{\alpha}\times(\boldsymbol{\alpha}\times\boldsymbol{r}) + \dot{\boldsymbol{\alpha}}\times\boldsymbol{r} + g(|\boldsymbol{r}_C+\boldsymbol{r}|)(\boldsymbol{r}_C+\boldsymbol{r}) - g(r_C)\boldsymbol{r}_C = 0 \\ \boldsymbol{r}(t_0) = \Delta\boldsymbol{r}, \qquad \dot{\boldsymbol{r}}(t_0) = \Delta\boldsymbol{v} \end{cases} \tag{22}$$

Proposition 3: The Deputy law of motion with respect to the LVLH reference frame associated to the Chief is modeled by

$$\boldsymbol{r} = \mathbf{R}_{-\boldsymbol{\alpha}(t)}\boldsymbol{r}_D - \boldsymbol{r}_C \tag{23}$$

where

1) $\mathbf{R}_{-\boldsymbol{\alpha}(t)}$ represents the rotation tensor with angular velocity $-\boldsymbol{\alpha}$ given in Eq. (21), $\boldsymbol{\alpha}$ represents the angular velocity of the rotating frame OPQR associated to the Chief.

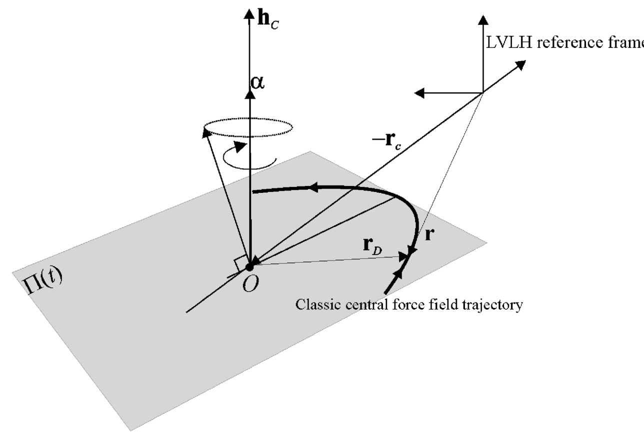

Fig. 1 Geometric vizualization of the relative orbital motion.

2) $\boldsymbol{r}_D$ is the solution to the initial value problem:

$$\ddot{\boldsymbol{r}} + g(r)\boldsymbol{r} = \mathbf{0}, \qquad \boldsymbol{r}(t_0) = \boldsymbol{r}_C^0 + \Delta\boldsymbol{r}, \\ \dot{\boldsymbol{r}}(t_0) = \boldsymbol{v}_C^0 + \Delta\boldsymbol{v} + \boldsymbol{\alpha}_0\times\Delta\boldsymbol{r} \tag{24}$$

3) $\boldsymbol{r}_C$ is the position vector of the Chief with respect to the OPQR reference frame (vector $\boldsymbol{r}_C$ models a rectilinear motion with respect to LVLH).

Remark 2: The geometrical visualization allowed by Proposition 3 is the following: the relative motion of the Deputy with respect to the Chief may be seen in the LVLH frame as 1) a classic central force field motion in a variable plane $\Pi(t)$, $t \geq t_0$; 2) a precession of plane $\Pi(t)$ with angular velocity $-\boldsymbol{\alpha}$ around the attraction center O; and 3) a rectilinear translation of plane $\boldsymbol{\Pi}(t)$ described by $-\boldsymbol{r}_C$ (see Fig. 1).

Remark 3: One may remark that the motion modeled by Eq. (23) is also a composition between a Foucault Pendulum-like motion (described by $\mathbf{R}_{-\boldsymbol{\alpha}(t)}\boldsymbol{r}_D$) and a rectilinear motion (described by $-\boldsymbol{r}_C$). It follows that this interesting property may be applied to any relative motion in a central force field. The problem is in fact reduced to the study of the classic inertial motion in a central force field.

Conclusions

The paper offers a closed-form expression for the solution to the relative spacecraft motion under the influence general central gravitational field. The advantage of the tensorial approach vs the classic matrix approach is that it does not depend on the type of coordinates that are chosen in the LVLH frame (Cartesian, cylindrical, spherical). The matrix approach that is present in most papers is strictly related to a set of Cartesian coordinates. It is shown that the relative motion is a composition of a Foucault Pendulum-like motion and a rectilinear one.

References

[1] Schaub, H., "Incorporating Secular Drifts into the Orbit Element Difference Description of Relative Orbits," AAS Paper 03-115, 2003.
[2] Balaji, S. K., and Tatnall, A., "Precise Modelling of Spacecraft Relative Motion for Formation Flying Spacecraft," IAC Paper 03-A.P.15, Sept.–Oct. 2003.
[3] Gurfil, P., and Kasdin, N. J., "Nonlinear Modeling of Spacecraft Relative Motion in the Configuration Space," *Journal of Guidance, Control, and Dynamics*, Vol. 27, No. 1, 2004, pp. 154–157.
[4] Gurfil, P., and Kholshevnikov, K. V., "Manifolds and Metrics in the Relative Spacecraft Motion Problem," *Journal of Guidance, Control, and Dynamics*, Vol. 29, No. 4, July–Aug. 2006, pp. 1004–1010.
[5] Gurfil, P., and Kholshevnikov, K. V., "Distances on the Relative Spacecraft Motion Manifold," AIAA Paper 2005-5859, Aug. 2005.
[6] Condurache, D., and Martinuşi, V., "Kepler's Problem in Rotating Reference Frames, Part 1: Prime Integrals, Vectorial Regularization," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 1, 2007, pp. 192–200.
[7] Condurache, D., and Martinuşi, V., "Kepler's Problem in Rotating

Reference Frames, Part 2: Application to Relative Satellite Motion," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 1, 2007, pp. 201–213.

[8] Marsden, J. E., and Ratiu, T. S., *Introduction to Mechanics and Symmetry: A Basic Exposition of Classical Mechanical Systems*, 2nd ed., Springer, New York, 1999, pp. 249, 291.

[9] Battin, R. H., *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA, Reston, VA, 1999, pp. 503–504.

[10] Gurfil, P., "Analysis of J2-Perturbed Motion using Mean Non-Osculating Orbital Elements," AIAA Paper 2004-4853, Aug. 2004.

[11] Condurache, D., "New Symbolic Methods into the Study of Dynamic Systems" (in Romanian), Ph.D. Thesis, Technical University "Gh Asachi,", Iaşi, Romania, 1995.

[12] Humi, M., "Fuel-Optimal Rendezvous in a General Central Force Field," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 1, 1993, pp. 215–217.